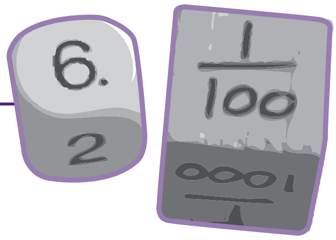
# DECIMATS:

## Helping Students To Make Sense Of Decimal Place Value

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introduces "decimats" and describes how they can be used to make sense of decimal size and decimal place value.

### Background

A considerable body of research exists on students' understanding of decimal fractions and the prevalence and persistence of common misconceptions related to this understanding (Steinle & Stacey, 2004). Much of this research knowledge has been inferred from responses to pen and paper tests. Data obtained from interviews are consistent with these findings but also highlight the strategies that are in harmony with conceptual understanding of the relative size of decimals (Roche, 2005; Roche & Clarke, 2004). Conceptual understanding includes knowledge of decimal place value and connections to fractional language for describing decimals.

Results from major studies such as the National Assessment of Educational Progress (NAEP) (Wearne & Kouba, 2000) in the United States and the Concepts in Secondary Mathematics and Science (CSMS) (Brown, 1981) in Great Britain also indicated that decimals create great confusion for many students.

Results from a multiple choice assessment item (see Figure 1) that was used in the Year 7 Achievement Improvement Monitor (AIM), a standardised test administered in all Victorian schools prior to national testing, provide interesting data.

Figure 1. AIM item Year 7, 2007.

This task would be relatively straight forward for a student who understood the "size" of 0.97. A student who understands 0.97 to be "nearly one" would quickly determine that 30.1 × 0.97 was close to 30. However, if the student did not have this understanding of the size of the decimal, they may feel compelled to attempt the multiplication first before choosing—not a trivial task. Therein lie a number of procedural difficulties that may account for the distractors (0.003, 3, and 3000). Only 63.0% of Year 7 students correctly chose 30. Interestingly, 19.3% chose "3000", presumably ignoring the decimal point and multiplying 30 × 97 and therefore treating 0.97 as a whole number. This misconception (that a decimal can be treated like a whole number) is very common for primary students and has been found still to exist with some students in Year 10 (Steinle & Stacey, 2003).

## Possible models for decimals

A range of representational tools has been proposed to assist students as they develop understanding of decimal place value. These include the hundred square, decipipes, Linear Arithmetic Blocks (LAB) (Helme & Stacey, 2000) and Multibase Arithmetic Blocks (MAB). The hundred square has the disadvantage of only discretely representing hundredths. MAB, commonly used for whole numbers, can be reconceptualised to introduce decimals, where the "block" is now considered as a "one," the flat as a tenth, and so on. However, it has been noted by teachers that some children find making this conceptual leap difficult. Money is sometimes proposed as a helpful context, and although it is a mathematically appropriate context in which to discuss decimals, it has a major disadvantage. The coins themselves do not "model" the difference in size between tenths and hundredths and the coins could easily give the appearance of whole numbers (e.g., so many dollars and so many cents) which could affirm a student's misconception that decimals can be treated like two whole numbers separated by a decimal point.

## The Decimat and its use in "Colour in Decimats"

I now describe an activity that helps students to make sense of decimal size and decimal place value and which encourages the use of fractional language to describe decimals. The activity involves the use of a Decimat (New Zealand Ministry of Education, 2004; Wright, 2004) which is a proportional model for representing the size of decimal fractions and makes explicit the "ten-ness" of our base ten place value system (i.e., ten hundredths make one tenth). The Decimat was developed originally in New Zealand.

I developed the game after seeing the benefits in developing student understanding of fractions through the game Colour in Fractions (Clarke & Roche, in review; Clarke, Roche & Mitchell, 2008). The rules of Colour in Decimats are very similar to those of Colour in Fractions, with one important difference. In Colour in Fractions, the students are not permitted to split a "brick" in the wall to make fractions of other sizes. In Colour in Decimats, it is permitted and essential to partition the tenths or hundredths, but only into ten equal, smaller parts.

## Introducing the Decimat

The Decimat (see Figures 2 and 3) consists of a large rectangle that represents "one" or "one whole". The rectangle is then partitioned into ten equal parts, creating tenths. One of these tenths is further partitioned into ten equal parts creating hundredths, and one of the hundredths is partitioned into ten parts creating thousandths. Unlike the hundred square which is also a proportional model for representing decimals, the Decimat extends to thousandths and in some cases can be used to represent decimals that are even smaller. The structure of the Decimat allows students to envisage further partitioning, creating tens of thousandths and further, even though it may be physically too difficult to make the lines distinguishable.

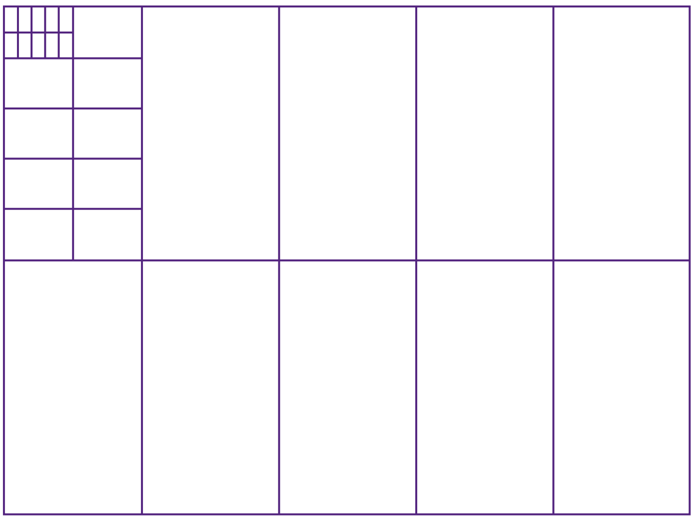

Figure 2. A blank Decimat.

## Colour in Decimats: The rules

Students have 2 die: a normal six-sided die (although I prefer those with 6 digits rather than dots) and a blank die that has faces labelled $\frac{1}{10}, \frac{1}{100}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{1000}, \frac{1}{1000}$ respectively, a Decimat gameboard including a recording table (see Appendix 1), and coloured pencils or textas.

Player A rolls the two dice and then shades the product of the two numbers displayed using a coloured pencil or texta. For example, rolling a "3" and "$\frac{1}{100}$" creates $\frac{3}{100}$. She/he then records what has been rolled as a fraction and as a decimal, using the same colour. In the last column, Player A records (as a decimal) "how much is shaded altogether?" at the completion of that turn. As the game proceeds, this column records the cumulative total of how much is shaded to that point of the game. Player B then rolls the two dice and continues the game using her/his own Decimat gameboard. Players take turns in this way. Each player records how much of their Decimat is shaded after her/his turn. A new colour is chosen at each roll of the dice. This enables the student (and teacher moving around observing and questioning students) to match the recorded roll with what has been shaded, checking for possible errors.

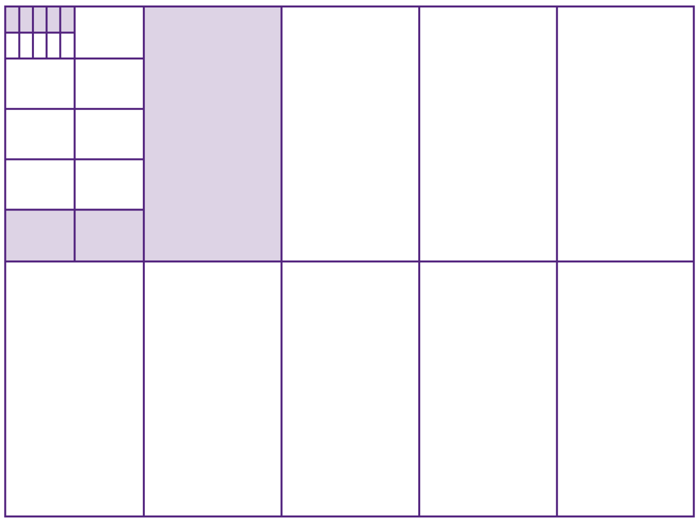

Figure 3. A shaded Decimat (showing 0.125).

It is permitted for tenths/hundredths to be further divided into hundredths/thousandths, if more hundredths/thousandths are required. For example, if a player has shaded all of their hundredths and they then roll 4 hundredths then the player can "cut" a tenth into ten equal pieces (therefore creating 10 hundredths) and shade four of these. This helps to reinforce the relationships between tenths, hundredths and thousandths.

The first player to reach one (by shading their complete Decimat) or the player closest to one (by shading more of their Decimat) after an agreed amount of time has elapsed, is the winner. If the product of the roll is greater than the remaining unshaded part of the Decimat, then the player misses a turn.

## A sample game

Below is an example of a game that has begun between Player A and Player B.

Over 3 turns, Player A rolls $\frac{3}{10}$, $\frac{6}{100}$ and another $\frac{6}{100}$. After 3 turns, 0.42 or $\frac{42}{100}$ of the Decimat is shaded (Figures 4-6).

Over 2 turns, Player B rolls $\frac{6}{1000}$ and $\frac{5}{1000}$. After 2 turns, 0.011 ($\frac{11}{1000}$, or $\frac{1}{100}$ and $\frac{1}{1000}$) of the Decimat is shaded (Figures 7 and 8).

Note that on Player A's third turn, she/he has chosen to create ten more hundredths from a new tenth, and then shade six of these. They could have chosen to shade the remaining four hundredths (including the ten thousandths) and then create ten more hundredths and shade two of these. By choosing to create more hundredths, rather than shade the remaining hundredths first, the thousandths are available for future rolls. Students are permitted to make choices like these, because they still have to total up all the "bits" as they record the progressive total through the game.

Figure 4. Player A first roll ($\frac{3}{10}$).

Figure 5. Player A Second roll ($\frac{6}{100}$).

## Introducing the Decimat before the game

Prior to playing the game, I usually spend some time with the class making sense of the model. Initially, I show them a Decimat containing only tenths with one shaded and discuss how much of the whole rectangle is shaded and how we might record this as a fraction and as a decimal. I then present a Decimat with hundredths included and one of the hundredths shaded and continue the discussion; and so on with thousandths. I have some large laminated Decimats that are partially shaded and the students discuss how we might write the shaded amount as a decimal (See Figure 9). Among the examples I include shaded Decimats, where the students need to record a decimal where there are no tenths and some hundredths shaded, so that they consider the importance of the zero as a placeholder. This initial discussion is important to help students make sense of the Decimat and how they might record "how much is shaded altogether?" as a decimal, as this is an important feature of the game they are about to play.

Figure 6. Player A third roll ($\frac{6}{100}$).

Figure 7. Player B first roll ($\frac{6}{1000}$).

Figure 8. Player B second roll ($\frac{5}{1000}$).


Figure 9. Recording the shaded amount as a decimal.

## Important features of the game "Colour in Decimats"

1. An important feature of this game is the use of colour to differentiate each turn. This makes it possible for the teacher or the student to check that what they have shaded matches what they have recorded as their roll. I have found that when students are first introduced to this game, errors can be common. It is helpful for the teacher to be roving and checking as the students are playing, and over time and with subsequent episodes with this game, there will be fewer errors. As with most games in mathematics, it is crucial to play it on a number of occasions to make the most of the experience in a mathematical learning sense.
2. The wooden die has $\frac{1}{10}$ on one face, $\frac{1}{100}$ on two faces, and $\frac{1}{1000}$ on three faces, written as fractional notation on the cube. This has been done intentionally. The fraction notation (rather than decimal) encourages the students to use fractional language (rather than "zero point one" or "zero point zero one", etc). The use of fractional language helps to create a strong link between the fraction notation, the decimal notation and the decimal place value. For example, if I roll "three hundredths," then I need to record this as a fraction and as a decimal, thereby connecting $\frac{3}{100}$ to 0.03. Also, the abundance of thousandths on the die (rather than 2 tenths and 2 hundredths and 2 thousandths) increases the necessity to create thousandths and therefore explore the ten-ness of the relationship between tenths and hundredths and thousandths.
3. Providing opportunities for the students to further partition tenths or hundredths to create hundredths or thousandths, helps to make explicit the structure of our place value system (e.g., a tenth is ten times bigger than one hundredth).
4. When recording in the "How much is shaded altogether?" column, it is anticipated that the students will count the number of tenths, hundredths, etc., that are shaded. However, as partitioning occurs, this becomes more difficult. The student is then required to make sense of, for example, a Decimat that has 2 tenths, 12 hundredths and 16 thousandths shaded. This affords opportunities for students, with appropriate support from peers and possibly the teacher, to regroup and rename, such that 10 of the hundredths makes one tenth, and 10 of the thousandths makes one hundredth, thus totalling 3 tenths, 3 hundredths and 6 thousandths or 0.336.
5. The recording table provides evidence of the student's capacity to move appropriately between fraction notation and decimal notation, to match these to an appropriate amount of shaded area on the model, as well as facility with the addition of decimals.
6. As the game is not intended as a "testing" situation, the place value grid has been included to help remind students which "place" after the decimal point represents tenths, hundredths, thousandths, respectively.
7. I recommend the game for Year 4 to 8 students, however the game can be adapted for younger students by removing the thousandths from the gameboard and from the die.
8. In order for the game to have maximum effect, it needs to be the basis of a whole lesson each time it is played, and not just a "warm up" (Bragg, 2006). Careful debriefing with questions like, "if you played the game again, what would you do differently?" or "what did you learn mathematically from the game today that you didn't know when you walked in the door?" can help to maximise the effect of the game.

## Conclusion

I encourage teachers to use this game to introduce students to the Decimat as a model for representing decimals. Once the students are familiar with this model, it can then become a tool for making sense of decimals. When students exhibit some misconception or misunderstanding about the relative size of two decimals, they could shade in two blank Decimats and compare the area shaded.

## Acknowledgements

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