Place Value underpins much of what we do in number.

JEANETTE BERMAN describes some simple tasks that may be used to assess students’ understanding of Place Value.

When exploring the learning needs of young children in literacy we look at how well they understand and can manipulate our system of written language. We can do the same in numeracy by assessing understanding and manipulation of our place value number system. This set of tasks, the Six Tasks of Place Value (SToPV), takes five minutes to administer and can give you direct insight into a student’s understanding of our number system and therefore what teaching is needed. It can be administered to students generally from Year 3, as this is when this understanding is expected to be achieved by most students (Board of Studies, NSW, 1998). This is also when some students’ conceptual understanding and procedural learning of arithmetic becomes less integrated and can contribute to mathematical learning difficulties (Hopkins, 2011). The highly integral nature of understanding place value numeration as it is applied in arithmetic and problem solving is acknowledged (Kamii, 1989; Steffe & Cobb, 1988; Wright, 1996) as is the range of ages at which students can establish this understanding, if they do (Cockcroft, 1982). This set of tasks therefore aims to support teachers in validly exploring their students’ understanding so they can ensure this crucial conceptual aspect of mathematical learning is well established.

The Six Tasks of Place Value (SToPV) were derived from tasks used by cognitive
researchers in this domain of mathematical learning. Sierink and Watson (1991) identified seven tasks that could represent the domain of learning—a compilation of tasks from previous researchers and one they designed. Berman (2001) reduced these to six tasks. A conservation of quantity task, that was thought to be a prerequisite to the cognitive understanding of place value, was found through Rasch analysis to be unnecessary and so was removed from the set. The remaining six tasks assess part–whole representation, counting, multi-digit reading and writing, and the role of zero as a placeholder in numbers. Each task has a set of levels of response against which to match student responses. Analysis of the responses to the six tasks can result in a determination of the stage of understanding of place value of two digit numbers. The three stages are emergent (preliminary concepts and skills), construction (developing concepts and skills), and established (consolidated concepts and skills).

The SToPV is designed for administration within an individual interview, similar to other recent task-based assessment procedures such as the Schedule for Early Number Assessment (SENA, NSW Department of Education and Training, 1998). The SToPV can be administered by teachers and other assessment professionals, and only requires a small collection of materials, all of which are common in primary classrooms or homes.

The materials

You will need:

• some base ten blocks
• 40 unit blocks (shorts)
• eight tens (longs)
• 26 counters of some sort
• 60–100 beans or other small objects for counting
• 10 blank cards upon which to write or print numbers
• a pantry item, such as a biscuit packet, that has a use-by date that contains a two digit number with zero in the first column, e.g., 06 Apr.

Administration and interpretation

Task 1

Give the student the base ten blocks (40 shorts and 8 longs). Say: “Use these counting blocks to build 52.” When the blocks have been arranged to show 52, ask, “How do you know that is 52?” Determine which level of response the arrangement and explanation fits in reference to these four levels of performance and record it on the record form.

Level 4 (achieved): The student performed the construction of 52 using five longs and two shorts and explained that the longs were tens and the shorts were ones and so five tens and two shorts make up 52.

Levels 3 and 2 (partial): The student can construct the number but does not provide a correct explanation. A lower level partial response is correct construction after an attempt with shorts only.

Level 1 (unsuccessful): the student was not able to select five longs and two shorts to make 52. The number of blocks does not allow for total use of shorts to make the number.
Task 2
Place a pencil and paper on the desk and a pile of 26 counters. Say: “Count these and write down how many there are.” When the student has written the number on the paper say, “Sort them into groups of four.” Then circle the 6 in 26 and ask, “What does this part of your 26 have to do with how many counters you have?” Circle the 2 in 26 and ask, “What does this part of your 26 have to do with how many counters you have?” Determine the level of response and record it on the record form. There are four levels of response for this task.

**Level 4 (achieved):** An explanation that the 2 refers to the number of tens and the 6 is the number of units or ones.

**Level 3 (developing):** An explanation that the 2 means twenty and the 6 means six.

**Level 2 (face value):** An explanation that the 2 means the 2 counters (left over) and the 6 means six groups (as arranged).

**Level 1 (unsuccessful):** the student does not know what they mean, or the student wrote the number incorrectly, or gave a general response such as “to make it twenty-six.”

This task is controversial in the field of mathematics education since it deliberately provides a distractor towards the face value response at level 2. The groupings of four counters, that results in six groups of four counters and two left over, can suggest to the student that this is what is being asked. This shows that the student’s understanding of tens and ones is still being constructed. However, a student who has developed understanding of place value of two digit numbers will not be distracted by this arrangement. You need to be conscious that a student may feel tricked and you can then combine all the counters and ask the question again to see if that allows the student to provide a response at the higher level.

Task 3
Have available a large number of beans or small objects, between 70 and 100, for example, 84. Place the pile of beans in front of the student and say, “How many beans do you think there are? Count them.” Record the response. Now say, “Count them.” Note what strategy the student uses and if it was not by tens, then say, “Count them again by tens.” Record the level of response and any comments made by the student.

**Level 4 (achieved):** Counts groups of ten with conservation of the whole, fluently.

**Level 3 (developing):** Counts groups of ten and joins them in process, showing that the ten is an entity.

**Level 2 (developing):** Places the beans in groups of ten, but does not tell how many there are altogether. When asked “How many beans are there?” gives the number of piles rather than the number of beans, or counts into piles of ten but is still really counting by ones.

**Level 1 (unsuccessful):** The student does not know what this means, or they count each bean as ten.

Task 4
Show the student a use-by date that has a two digit number with the zero in the tens place such as 05 Oct. Point to the 05 and
ask, “What number is this?” Seek further explanation with: “Why?” and “What does the nought (zero) do?”

Determine the student’s level of response in reference to these levels.

**Level 3 (achieved):** States that the number is 5 and the 0 (zero) is a placeholder for the number of tens.

**Level 2 (developing):** States that it is not 50, but is not sure what it is.

**Level 1 (unsuccessful):** States that the number is 50 or no indication that the zero means anything significant.

**Task 5**

Provide a pencil and paper, and say, “Write these numbers: 3, 6, 19, 83, 109, 1607, 3045, 6572.” Leave enough time for the student to write each number. It may be useful to provide boxes for each number on the paper to keep them separate and easy to read.

Record which number of digits the student is able to write correctly.

**Task 6**

You will need ten cards with these numbers written on them. You can use cardboard with computer-generated numbers or can handwrite them. Show the number cards one at a time to the student, asking, “What number is this?” with the first card. Record what is said if it is not correct. The numbers have been selected so that they require knowledge of the naming of numbers with and without zeros.

**Stage of understanding**

From the responses to each of these six tasks, you can now make a determination about where the student is in their construction of understanding of place value against the developmental framework adapted from Ross (1986). You can plot the response levels on this map and then refer to the definitions of each stage to decide which stage the student is demonstrating.

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<th>Task</th>
<th>Non-standard partitioning</th>
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**Stage of understanding**

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<th>Understanding</th>
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Understanding
Students who demonstrate understanding are those who provide the highest level response to each of the tasks and provide explanations that numbers can be divided into tens and ones, irrespective of how the quantities are arranged. They would also be able to explain the role of zero in the notation of numbers and would be showing skills in reading and writing three and four digit numbers.

Construction
Students in this stage will provide evidence of understanding but this will not be consistently demonstrated. They will have some lower level responses. They may be able to read and write all the numbers, and count by tens, but not be able to explain the role of zero satisfactorily.

Emergent
Students in this stage can identify and label the tens and ones in a two digit number, but not explain the quantities they represent. Task 2 is designed to identify this level of understanding, through the grouping distractor that supports a face value interpretation of the digits.

Once you have determined the stage of understanding for a student, you can target your teaching to encourage further development along this conceptual pathway. The levels of response for these tasks plot future learning and therefore teaching.

Conclusion
The SToPV will allow you to spend five minutes in an individual task-based interview with students to determine clearly where they are in their construction of understanding of the number system. This understanding is vital to the development of effective skills in arithmetic and needs to be firmly established before they resort to procedural learning rather than trying to make conceptual sense of number. The SToPV is firmly based in cognitive research in this area of student learning and can be administered with everyday classroom materials. It allows for clear definition of a student’s understanding and also a pathway for further teaching and learning.

References

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